**Equivalent Circuit Analysis of Interior Permanent Magnet Synchronous Motor Considering Magnetic saturation**

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**Abstract**

Generally, design of the parts of an automobile is restricted by space especially components assembled in the engine room. Therefore traction motor for hybrid electric vehicle is designed by interior permanent magnet synchronous motor (IPMSM). The IPMSM has high power density and efficiency because it generates not only magnetic torque but also reluctance torque. However, IPMSM is difficult to design due to magnetic saturation and lots of analysis methods are required. Equivalent circuit analysis (ECA) is one of the design methods of IPMSM and it is generally used for characteristics analysis. Using the ECA, we can calculate the characteristics quickly. However, ECA can not estimate line current exactly because of saturation of magnetic core and flux weakening control. In order to perform the ECA exactly, calculation of parameters such as phase resistance, \(d\)- and \(q\)-axis inductance and no-load linkage flux are important. The phase resistance is simply calculated by using linear equation which is geometrical problem. However, \(d\)- and \(q\)-axis inductance is nonlinear problem which is varied with current level and current angle. Therefore, nonlinear finite element analysis is necessary. Conventional ECA is considered only \(d\)- and \(q\)-axis inductance profile but no-load linkage flux is considered as constant value. However, no-load linkage flux of IPMSM is also varied according to current level and current angle similar with \(d\)- and \(q\)-axis inductance profile. Therefore, no-load linkage flux profile which is calculated case of current level and current angle should be considered in the ECA. In this paper, the calculation method of no-load linkage flux profile is introduced firstly and then characteristic of specific model is analyzed with proposed method. The result of line current from improved ECA is compared with experimental result and result of conventional ECA.

**Keywords:** Equivalent circuit analysis (ECA), Interior permanent magnet synchronous motor (IPMSM), No-load linkage flux profile

**1 Introduction**

The traction motor, inverter and battery are mainly additional parts to compose hybrid electric vehicle (HEV) and these components require small volume and high efficiency. In order to reduce the volume and improve the efficiency of traction motor, interior permanent magnet
synchronous motor (IPMSM) is generally applied to traction motor for HEV because permanent magnets are embedded in the rotor core, IPMSM generates both magnetic torque and reluctance torque by the rotor saliency. In order to analyze characteristics of IPMSM according to variation of parameters, equivalent circuit analysis (ECA) usually applied despite of low accuracy. A lot of researcher is performed to improve accuracy of ECA such as consideration of the core-loss resistance or $d$- and $q$-axis inductance profile according to amplitude of line current and current angle [1]-[4]. However, these methods also include error because of no-load linkage flux input as a constant value regardless of line current and current angle. From this reason, line current has little difference between measured values and calculated value by ECA especially flux weakening region.

This paper deals with the improvement of the ECA by considering variation of no-load linkage flux which is changed by magnetic saturation and armature reaction. Lastly, line current calculated by improved ECA, conventional ECA and measured value are compared to verify the analysis method presented in this paper.

2 Equivalent circuit analysis

Equivalent circuits for IPMSM based on a synchronous $d$-$q$ reference frame including iron loss are presented in Figure 1. The mathematical model of the equivalent circuits is given as following equation [3].

Iron loss is considered by equivalent resistance $R_c$, and the $d$- and $q$-axis voltages, effective torque and definition of current are given by equation (1), (2), (3) and (4) respectively

$$
\begin{align*}
\begin{bmatrix} v_d \\ v_q \end{bmatrix} &= \begin{bmatrix} R_d & \frac{i_{sd}}{L_{sd}} \\ \omega L_{q} & R_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} + \begin{bmatrix} 1 + \frac{R_c}{R_d} \end{bmatrix} \begin{bmatrix} v_d \\ v_q \end{bmatrix} \\
+ p \begin{bmatrix} L_{sd} & 0 \\ 0 & L_{sq} \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} \\
\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} &= \begin{bmatrix} 0 & -\omega L_{q} \\ \omega L_{d} & 0 \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} 0 \\ \omega \psi_a \end{bmatrix}
\end{align*}
$$

(1)

$$
T = P_n \left( \psi_d i_q + \left( L_d - L_q \right) i_{sd} i_{sq} \right)
$$

(2)

$$
i_e = \sqrt{i_d^2 + i_q^2}
$$

(3)

where, $i_d$ and $i_q$ is $d$- and $q$-axis current. $i_{sd}$ and $i_{sq}$ is core loss current. $v_d$ and $v_q$ is $d$- and $q$-axis input voltage. $R_d$ is a phase resistance and $\psi_a$ is linkage flux per single pole. $L_d$ and $L_q$ is $d$- and $q$-axis inductance, respectively. $P_n$ is the number of pole pair and $i_e$ is line current.

3 Armature reaction

Generally, synchronously rotate reference is applied to describe vector diagram of IPMSM. Conventional ECA is analyzed with using constant value of linkage flux per single pole but it is different from real phenomenon because of nonlinear characteristics of magnetic circuit. In order to improve ECA, armature reaction and magnetic nonlinearity is considered in the ECA. Figure 2 briefly shows the variation of flux density per single pole pair. Line A in the Figure 2 is field flux density in the air-gap which is generated by PM and Line B is air-gap flux density that is generated by only armature current. The Line C shows resultant flux density with linear condition. In the Line C, half of field flux density is demagnetized and the other half of field flux density is magnetized by armature reaction. If magnetic circuit has a linear characteristic like electric circuit, R.M.S value of air-gap flux density is always same irrespective of armature current. However, magnetic circuit has a nonlinear characteristic. Therefore, the half of magnetized part of flux density is saturated as shown the Line D. It means that linkage flux form PM is varied according to input current level and current angle. In order to consider the nonlinear characteristics of IPMSM above mentioned, linkage flux generated by PM may be calculated by FEA according to case of current level and current angle.
4 Analysis model

Analysis model is designed for HEV traction motor. The specification of an analysis model is shown in Table 1. The battery voltage is 155 V DC and R.M.S. value of current limit is 200A. The maximum torque, 105 Nm is produced by maximum torque per ampere (MTPA) control at the low speed, from 0 to 1200rpm. Flux weakening control is applied to maintain constant power 15kW until 6000rpm.

5 Calculation of linkage flux from PM

The constant value of no-load linkage flux $\Psi_a$ is considered in conventional ECA. However, linkage flux $\Psi_a$ generated by PM is varied because of magnetic saturation which is changed by line current level and current angle. Therefore no-load linkage flux $\Psi_a'$ which is considered magnetic saturation at load condition may input ECA with form of look up table as a function of line current and current angle as shown in Figure 3. The $\Psi_a'$ of analysis model is calculated by finite element analysis.

6 Improved ECA

Overall process of characteristics analysis is shown in Figure 4. In the conventional ECA, the $d$- and $q$-axis inductance profile is input in the conventional ECA to consider nonlinearity of magnetic circuit. On the other hand, analysis process of improved ECA is similar with conventional method except linkage flux $\Psi_a'$. The $d$- and $q$-axis inductance is calculated by equation (5) [3].

$$L_d = \frac{\Psi_a - \Psi_a \cos \alpha}{i_d}, \quad L_q = \frac{\Psi_a \sin \alpha}{i_q} \quad (5)$$

Improved ECA can consider the variation of $\Psi_a'$ according to current level and current angle. Two analysis stages are required to analyze linkage flux $\Psi_a'$. First, load analysis is performed using by FEA with input current and current angle. Second, no-load analysis is performed without current source. In the analysis of stage two, magnetic saturation information calculated in first analysis stage is considered for calculation of $\Psi_a'$. The value and angle of linkage flux $\Psi_a'$ has a difference with $\Psi_a$ as shown in Figure 5. Therefore torque equation (3) can be modified as follow equation (6).

$$T = P_a \left( \Psi_a' i_{q0} \cos \chi + \left( L_d - L_q \right) i_d i_{q0} \right) \quad (6)$$

Table 1: Specification of analysis model

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
<th>Unit</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage</td>
<td>155</td>
<td>V DC</td>
<td>DC link</td>
</tr>
<tr>
<td>Max. torque</td>
<td>105</td>
<td>Nm</td>
<td>@1200rpm</td>
</tr>
<tr>
<td>Output Power</td>
<td>15</td>
<td>kW</td>
<td>Max.</td>
</tr>
<tr>
<td>Current limit</td>
<td>200</td>
<td>A rms</td>
<td>Max.</td>
</tr>
<tr>
<td>Base speed</td>
<td>2000</td>
<td>rpm</td>
<td>MTPA</td>
</tr>
<tr>
<td>Max. speed</td>
<td>6000</td>
<td>rpm</td>
<td>Flux weakening</td>
</tr>
</tbody>
</table>

Figure 2: Variation of linkage flux according to armature reaction

Figure 3: Variation of linkage flux according to armature reaction

Figure 4: Overall process of presented ECA
Verification of improved ECA

In order to verify improved ECA, current mapping is performed. The equipment of experiment is shown as Figure 6. IPMSM produces not only magnetic torque but also reluctance torque. Therefore, IPMSM produces high torque better than surface mounted PM type motor with same volume. In order to achieve maximum torque from the IPMSM, current vector control is required. Therefore, current mapping is necessary to find optimal current and current angle. Line current which is measured as shown in Figure 7 is measured by current mapping. The line current calculated by presented method is compared with measured value to verify accuracy of presented analysis method. Figure 7 shows comparison of line current among the results from conventional ECA, improved ECA and measured value at the speed 1000, 3000, 6000rpm and each point of torque. The three results are almost same at the relatively low speed 1000rpm. However, results of improved ECA have good accuracy at the high speed 6000rpm better than result from conventional ECA. These results show that improved ECA can be used to predict line current more exactly better than conventional ECA during flux weakening control.

Conclusion

Many of analytic technique are required for characteristic analysis of IPMSM because of nonlinear characteristic. ECA which is one of generally used in design of IPMSM is not considering nonlinearity of IPMSM perfectly. In this paper, improved ECA is presented by considering linkage flux profile $\Psi'_a$. Presented method is helpful to predict reliable T-N-I curve of IPMSM. The next job is calculation of improved d- and q-axis inductance considering $\Psi'_a$. It will be also improving accuracy of ECA.

References


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